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ISLAMIC STAR PATTERNS

The geometrical ornament of Islam has fascinated Western observers for well over a century, and continues to provide material for numerous studies attempting to explain its nature in esthetic, mathematical, mystical, or even cosmological terms. The "geometrical" content of this ornament is obvious, but in spite of the fact that many works have been published on the construction and analysis of Islamic patterns,[1] none has delved very deeply into the subject, and no comprehensive work has yet appeared dealing in a comparative and systematic manner with the whole range of patterns, both geographically and historically.[2] There is clearly a need for a more scientifically based account of these patterns, although it is not so much the theoretical knowledge of the professional mathematician that is required for such a study, as the precise mathematical language and rigor which he might bring to bear on the subject. There is still no generally accepted terminology for the many different kinds of motifs used in Islamic geometrical ornament, nor for the methods of forming repeating patterns from them. In the absence of a definitive work along these lines, I will attempt here to show ways in which some of the simpler patterns might be developed from a number of elementary principles.

Opinions have differed as to the part played by mathematics in the genesis, development, and construction of the more complicated Islamic patterns.[3] Some have seen a gradual, convergent evolution from many different types of pre-Islamic ornament, culminating in the often bewildering complexities of the later, fully differentiated Islamic patterns. According to this view it was presumably the widening practical experience of skilled craftsmen which alone chose the paths taken at each level of increasing elaboration. Others, on the contrary, have seen clear evidence for the intervention of the professional mathematician in the design and invention of these patterns, or at least have ascribed to the artisans themselves a considerable knowledge of theoretical geometry and an ability to apply this knowledge to the development of new kinds of geometrical ornament.

We shall perhaps never have a final answer to this controversial question,[4] yet it seems that Western critics have seriously underestimated the ability of native craftsmen to retain large amounts of empirical knowledge on pattern design and construction in the absence of any understanding of the theoretical background which a professional mathematician might bring to bear on these problems.[5] It may be an advantage for a modern author to develop a systematic analysis of Islamic patterns in purely mathematical terms, but a knowledge of pure mathematics or geometry is unnecessary for those who wish merely to draw Islamic patterns or invent new ones. A theoretical background will often allow the artist to see a number of combinatorial possibilities more quickly than the use of trial-and-error methods, but it forms no substitute for true creativity. It is perhaps significant that a genuine application of mathematical insight to a systematic analysis of Islamic geometrical patterns reveals a far greater range of possibilities than were ever discovered by the Muslims themselves.[6]

A great deal of this ornament is immediately and unmistakably recognizable as "Islamic," and yet in its entirety it does not form such an easily distinguishable body of geometrical ornament. In fact the whole range of Islamic patterns represents an amalgam of many different styles, some simply adapted and absorbed from classical sources and from various cultures with which Islam came into contact during its early expansion. Since it is not possible to cover all types of Islamic patterns here, I will limit myself to an examination of certain of the more interesting and typical of them.

There is one class of geometrical patterns which Islam has made its own. This group comprises what one might term the "star patterns," since they include star-like motifs, linked or oriented according to certain precise rules to produce endlessly repeating two-dimensional patterns. The star patterns are unquestionably the most beautiful and intricate of all Islamic patterns, and they owe their beauty in no small measure to a high degree of symmetry at all levels. Indeed, the star motifs themselves invariably possess *n*-

fold rotational symmetry, *n* representing a range of whole numbers from 3 to almost 100. Although any pattern which repeats in two directions must of necessity pertain to one or another of 17 fundamentally distinct arrangements,[7] a classification by means of such symmetry groups is of little use in a detailed analysis of Islamic geometrical patterns. A given Islamic pattern will frequently employ a small number of precisely determined shapes, some of which become repeated within the pattern in many ways not allowed for under the four elementary operations, or isometries, of classical plane symmetry;[8] each of the latter must act on the whole two-dimensional plane, not merely locally on small parts of it. Similarly, although many motifs themselves possess higher than 6-fold rotational symmetry, they cannot form repeated centers of similar rotational symmetry in the plane as a whole, since the permitted centers can have no more than 2-, 3-, 4-, or 6-fold symmetry (such "rotocenters" are termed respectively diads, triads, tetrads, and hexads). On the surface of the sphere the restrictions are somewhat different, whereas in the hyperbolic plane virtually anything is possible.[9]

In their simplest form all Islamic geometrical patterns are examples of periodic tilings (or tessellations) of the two-dimensional plane, consisting of polygonal areas or cells of various shapes abutting on neighboring cells at lines termed the edges of the tiling, and with three or more cells meeting at points termed the vertices, or nodes, of the tiling. In general, the cells are not required to be convex polygons, nor the edges to be straight lines, and there is no restriction on the number of edges or cells meeting at each vertex.[10]

From the earliest times Islamic ornament adopted the widespread interlacing band form of linear decoration, whereby the original lines of the pattern are represented by straps or bands, executed in such a way as to give the impression of weaving alternately over and under one another. This style of ornament had its origins in antiquity, and must ultimately derive by imitation from various types of weaving, plaiting, or basketwork. As an artistic device it serves to give cohesion to a whole design. We may note that an interlacing-band style can only be achieved when the original tiling consists entirely of 4-way nodes, that is, four edges (and therefore cells) meet at every node.[11] Ideally, opposite angles at each node should be equal, which means that the four edges meeting at that node become a pair of lines intersecting at a crossover point. Patterns consisting entirely of 4-way nodes may be referred to as true *interlacing patterns*, whether or not they are drawn as interlacing bands.

Interlacing patterns have another important property: in their linear, as opposed to interlacing-band, form the cells of any pattern may be colored alternately in two modes—say, black and white—so that no two cells of the same mode meet at a shared edge. A chessboard is a familiar example. Strictly speaking, the possibility of a two-mode coloring is inherent in any tiling in which an *even* number of cells or edges meets at every node. *Non-interlacing patterns* contain at least some nodes which are not 4-way. If some of these are odd-numbered, then a two-mode coloring is no longer possible. The star patterns in general include examples from both interlacing and non-interlacing categories.

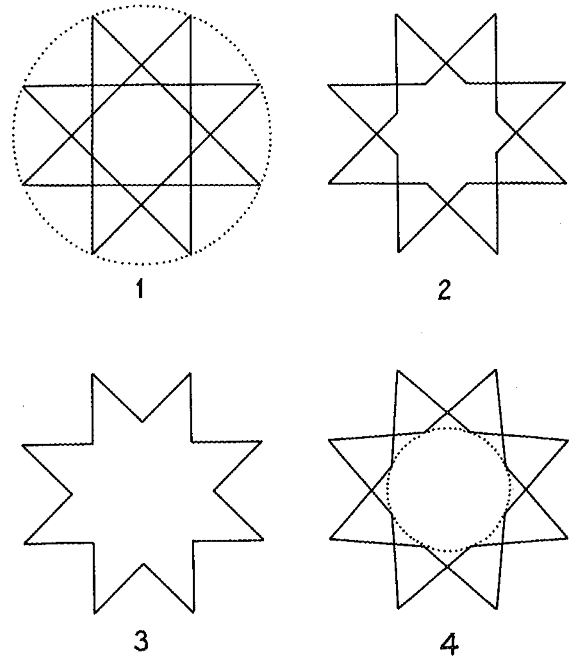
Islamic star motifs owe their beauty and regularity to a feature which they share with the regular convex polygons: all derive from sets of points equally distributed around the circumference of a circle. When pairs of adjacent points in any such set are joined by straight lines until a single circuit is completed, the result is a regular convex polygon. However, it is possible to continue joining up every other point, or every third point, and so on, to produce many beautiful star-like figures. In a loose sense these may all be termed "star polygons," although a star polygon, properly speaking, results only when all the lines so produced form a single circuit surrounding the center of the figure more than once. The set of points on the initial circumference comprises the *vertices* of the star polygon, but the sides of a star polygon intersect one another at various additional points, which are not counted as vertices, although they divide each side into a number of segments.

The earliest Islamic star motifs were based on a star polygonal construction, but complete star polygons were rarely used as ornamental motifs (usually in isolation, as medallions). Initially such a construction produces a space at the center of the figure, in the shape of a regular polygon (fig. 1). In authentic Islamic ornament this central space is usually transformed into a star-shaped area by the omission of one or more of the middle segments on all sides of the star polygon. Usually all but the last two segments at each end of a side are omitted, thus producing the typical Islamic star motif (fig. 2). This figure therefore consists of an inner cell, or central star, and a number of outer cells in the shape of kites. Occasionally all but the outermost segment are omitted (fig. 3), and we thereby arrive at the simplest form of a regular geometrical star.

Star motifs of these types can be distinguished by a concise notation, giving data on three quantities: the number of initial vertices; the method of joining up the vertices to produce the original star polygon (i.e., two by two, three by three, and so on); and the number of end segments remaining at each end of the sides of the star polygon. These three quantities can be represented by n, d, and s, respectively, and the complete symbol for the basic Islamic star as $(n/d)s$. Thus, the star shown in fig. 2 can be designated an (8/3)2 (this may be read as an "eight over three, two-segment star"). Obviously this notation, which is derived from the mathematical notation for star polygons, is applicable to any Islamic star constructed by drawing straight lines between pairs of points on a circle. Indeed, it may even be adapted to certain other types of construction, if we allow non-integral values for d.

Motifs based directly on star polygons (sens. lat.) are easily constructed using a single circle and a number of points equally distributed on the circumference of that circle. No other initial construction is necessary. Many later Islamic star motifs, however, are not derived in this way, although they may still consist of a central star and surrounding kites. In these later star motifs the slope of the lines forming the star is such that they cannot be constructed simply by joining pairs of points on the circumscribing circle of the motif. In these cases, one or more additional concentric circles are needed to determine the inner points of the star motif and hence to complete the lines (fig. 4). Islamic star motifs are of many different types, and the main varieties will be indicated below, but all include a simple star as a central cell. The precise metrical properties of any star motif may sometimes be arbitrarily chosen, but frequently depend on geometrical considerations concerning the relation of the motif to other elements in a pattern.

The symmetrical properties of any regular n-pointed star motif or n-sided regular polygon (n being any whole number greater than two) can be represented as a system of $2n$ radii diverging from the geometrical center of that motif. This figure is conveniently termed a *star-center*; it consists of n principal radii, through the main, outer points of the motif, and n secondary radii, alternating with them. The continuing invention of fundamentally new star patterns entails a search for all suitable arrangements of star-centers in the two-dimensional plane, but the number of possibilities is limited by the precise way in which each star-center must be oriented in relation to its nearest neighbors. In the method of linking star-centers that is preferred above all others in Islamic patterns, one radius from each star-center of a neighboring pair is coincident with the straight line joining their centers. In the case of a pair of star motifs, their centers and shared point of contact lie on a single straight line (fig. 5). This is therefore termed a *collinear link*.



Figs. 1-4. The formation of star-motifs from star polygons.

Another acceptable method of linking nearby star-centers is by having their nearest radii parallel, rather than collinear. In this case the straight line joining their centers no longer coincides with a radius of either star; such a relation may be termed a *parallel link* (fig. 6). Parallel links between star motifs are principally found in certain derivative patterns (described below) which are usually far more difficult to construct with a ruler and compass than those using collinear links. (It is probable that most of these derivatives were originally composed through rearrangements of certain elementary mosaic shapes—for example, cut tiles or pieces of wood inlay.)

Rules such as these for linking adjacent star motifs were never explicitly stated by Muslim artists, but were applied, probably unconsciously, in large numbers of varied patterns throughout Islam. Indeed, the employment of collinear links to join star- or flower-like motifs

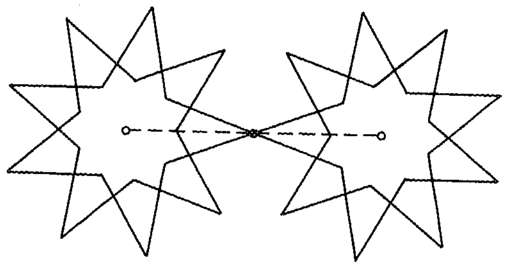


Fig. 5. A collinear link between two 9-pointed stars.

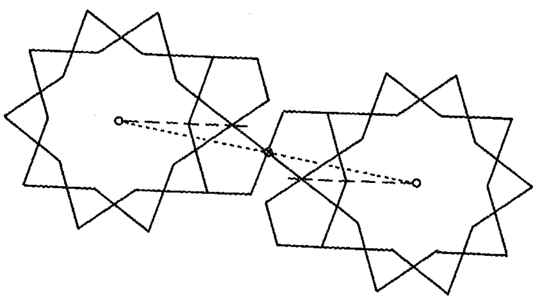


Fig. 6. A parallel link between two 10-pointed stars.

antedates their use in Islamic patterns by many centuries, and has probably always been felt instinctively to represent a more elegant method of pattern composition.

Many early forms of repeating patterns were based on simple fractional divisions of underlying grids of equilateral triangles or squares, both of which require the ruling of sets of parallel lines right across the patterned area. It was to such grids that the first designers of Islamic patterns turned to produce the earliest star patterns. The simplest procedure is to center a star motif on every vertex of the grid, with a radius equal to half the edge length of the grid polygons. The motifs are then oriented so that certain of their principal radii coincide with grid lines, and each pair of adjacent motifs meets at the midpoint of an edge of the grid. Thus, collinear links are automatically established. Since the triangular and square grids themselves have respectively 6- and 4-way vertices, the simplest motifs which can be used in this way will have 6 of 4 points. In fact, it should be obvious that motifs with any numbers of points which are integral multiples of these values can be similarly placed on the grid vertices.[12]

It is not easy to establish the exact date at which simple star patterns of this kind were first used in Islamic architectural decoration. Some attempts have been made to explain the historical and developmental origins of the first rectilinear star patterns,[13] but since curvilinear versions of many of these occurred even earlier, perhaps one should really try to explain the origins of the latter. In any discussion of historical origins, we must bear in mind that although it is still theoretically possible to locate the earliest surviving version of every distinct pattern, it cannot necessarily be assumed that any one of these really represents the first historical occurrence of that particular pattern. It is more than likely that most early examples of the art have long ago crumbled to dust. Nevertheless, sufficient material has survived from the eighth to the tenth centuries to give a tantalizing glimpse of what must have been an extremely rich fund of very early geometrical ornament. Recognizable precursors of the first star patterns started to appear in the Middle East as early as the beginning of the eighth century, in the form of open work window grilles in the Great Mosque at Damascus (715),[14] the palace of Qasr al-Hayr al-Gharbi in Syria (727),[15] and the palace of Khirbat al-Mafjar in Jordan (743).[16] Although some of these designs contain star-like elements, it is not always clear whether they represented distinct motifs in the artist's original conception, or whether they arose merely as residual spaces between groups of overlapping circles, circular arcs, or other shapes. In some cases, however (for example, that shown in fig. 7, of which almost identical versions survive from Qasr al-Hayr al-Gharbi and Khirbat al-Mafjar), we are probably correct in inter-

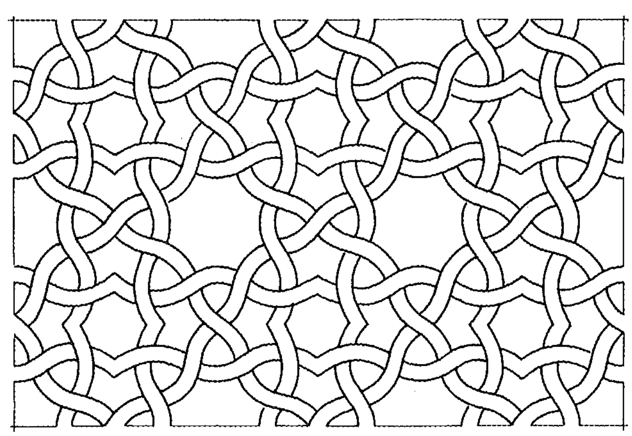


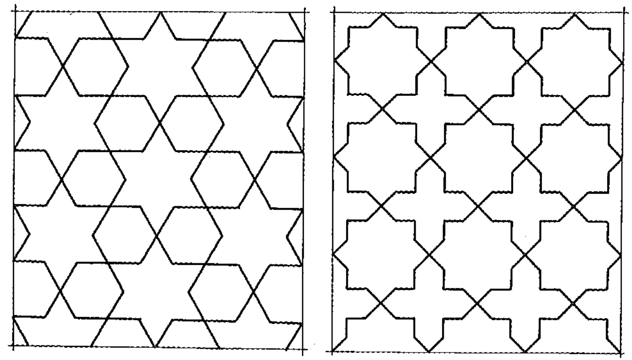
Fig. 7. A pattern from the eighth century with curvilinear 8-pointed stars.

preting the pattern as consisting essentially of an array of star motifs (in this case, of curvilinear 8-pointed stars) in contact.

Two of the earliest repeating patterns using simple rectilinear stars consisted of 6- and 8-pointed stars, respectively (figs. 8 and 9), both of which had their origins in classical antiquity.[17] In an Islamic context the star-and-cross pattern (fig. 9) first occurs in ninth-century Samarra;[18] a version of the pattern with 6-pointed stars occurs in the mosque of Ibn Tulun at Cairo (876-79);[19] while both can be seen in stucco from the palace of Madinat al-Zahra (936-76) in Spain,[20] and in al-Hakim's mosque in Cairo (1003).[21] However, the star-and-cross pattern probably appeared even earlier, since a rectilinear version occurs on one of the windows from Qasr al-Hayr al-Gharbi,[22] in which all line segments are extended as straight lines running in interlaced form right through the pattern. Another example of the pattern with 6-pointed stars occurs in an early-eleventh-century house excavated in Siraf.[23]

Thus by the end of the tenth century at the latest, patterns containing 6- and 8-pointed rectilinear stars were fairly widespread, and it probably required little intellectual effort on the part of the original artists to conceive the idea of incorporating simple stars with greater numbers of points in repeating patterns.[24] It is not until the second half of the eleventh century that examples survive, however. A pattern with 12-pointed stars superimposed on a triangular grid occurs on the earlier of the two Kharraqan tomb towers (1067-68)[25] in northwestern Iran (fig. 10). A related pattern, using the same (12/4)2 stars, but on the square grid (fig. 11), might be expected to have been discovered at about the same period, but no early examples appear to have survived. These patterns were later among the most widespread of all star patterns; they are found from Morocco to Central Asia, indicating in both cases very early discovery and dissemination. Similarly a pattern of (8/3)2 stars (fig. 12) had probably been discovered at this time, since the basic star itself was already in existence,[26] but again no early examples appear to have survived, though what could be regarded as a curvilinear version from the eighth century is illustrated in fig. 7.

The patterns developed so far immediately illustrate an important feature in the execution of nearly all Islamic star patterns. When the number of points in the stars is greater than the number of lines radiating from each node of the grid on which they are superimposed, there remains the problem of what to do with the



Figs. 8-9. Two of the earliest rectilinear star patterns.

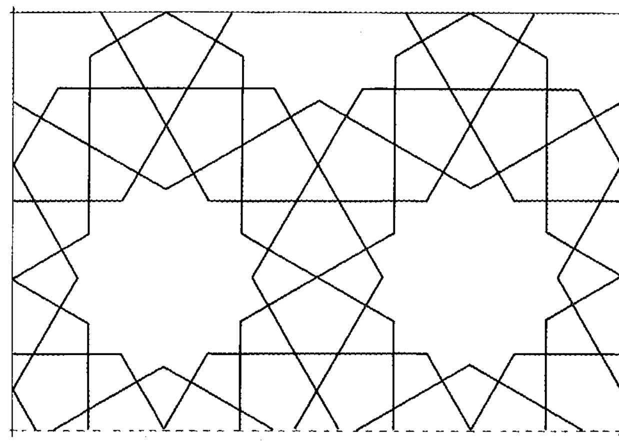


Fig. 10. 12-pointed stars on a triangular grid.

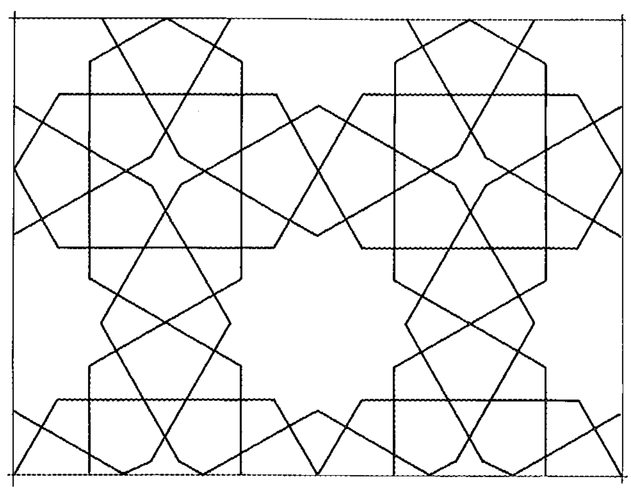


Fig. 11. 12-pointed stars on a grid of squares.

"free" vertices of each star, i.e., those not in direct contact with neighboring stars. The simplest and most elegant solution is to continue the edges of the stars through each free vertex until they meet similarly produced lines from neighboring stars. In the case of the square array of 8-pointed stars (fig. 12) this results in a small interstitial 4-pointed star, whereas in the case of the triangular array of 12-pointed stars (fig. 10) a small equilateral triangle is produced. In these two patterns each star contributes only a single free vertex in each grid polygon, and the total number of free vertices in each polygon is obviously equal to the number of sides in that polygon. The majority of star patterns obtain a greater measure of continuity between their constituent star motifs by bridging the space between free vertices in this way, although the artist is always at liberty not to do so if he wishes. The additional lines thus produced by joining up such free vertices may be referred to as constituting the *interstitial pattern*, since they occur in the residual space between groups of three or more neighboring stars. The greater the number of free vertices available in this space, the more complex is the interstitial pattern. The nature and symmetry of the elements in the interstitial pattern are clearly related to the symmetry of the pattern as a whole, as well as to the types of constituent star motifs and their precise construction.

Patterns using 10-pointed stars occur in the north dome chamber of the Masjid-i Jami^{c} at Isfahan (1088).[27] These "decagonal" patterns represent a departure from traditional methods of geometrical pattern design, although they result inevitably from a logical generalization of the principles developed so far. It is not immediately obvious how to form repeating patterns with 10-pointed stars, since neither of the grids considered so far is suitable. If the stars are required to remain in contact, using collinear links, however, it is found that there is only one simple arrangement possible, and this could easily have been obtained after a little trial and error (fig. 13). This pattern and the new grid which underlies it require a more detailed analysis (which will be given below) that leads to the formal discovery of many kinds of patterns. These include simultaneously two kinds of star motifs, including some of the most widespread of all Islamic star patterns. It is by no means suggested, however, that the original discovery of such patterns followed from the methods of analysis outlined here. Rather more elaborate patterns with 10-fold motifs occur on the later, so-called Victory Tower of Masud III at Ghazni in Afghanistan (probably dating from around 1100).[28] This structure also presents what appears to be the first occurrence in Islam of geometrical patterns incorporating 7-fold motifs (in fact these same patterns also include 20- and 14-pointed stars, respectively).

These "heptagonal" patterns, in which the proportions of the basic repeat are determined by angles which are multiples of 180°/7, are extremely difficult to work with. This difficulty is reflected in the paucity of examples throughout the whole range of Islamic geometrical ornament. About 6 percent of Bourgoin's collection[29] consists of true heptagonal patterns, and in this respect it is probably representative of Islamic patterns as a whole. Indeed, patterns using many other kinds of odd-numbered star-motifs are often very difficult to incorporate in repeating patterns, and were

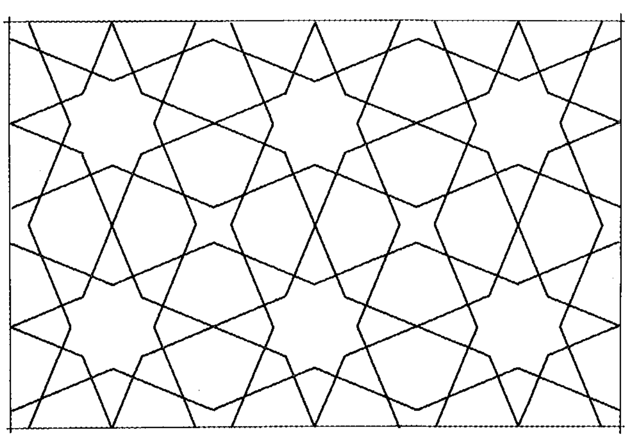


Fig. 12. 8-pointed stars on a grid of squares.

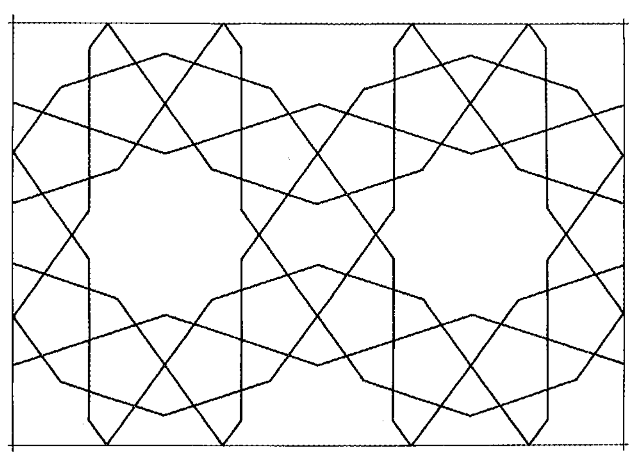
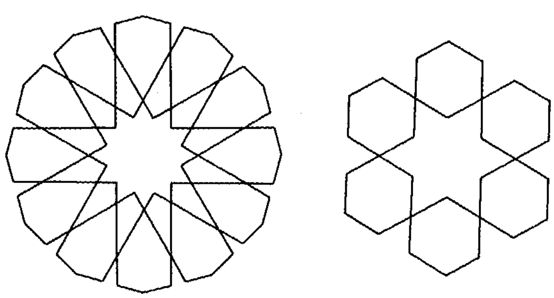


Fig. 13. 10-pointed stars of Type I on a grid of 72°/108° rhombs.

therefore seldom or never used. This is particularly so in the case of those motifs in which the numbers of points are prime numbers—say, 11, 13, 17, 19, and so on.

Before analyzing some of these new kinds of patterns, we must first consider the discovery and development of what is perhaps the most typically "Islamic" of all star motifs: the *geometrical rosette* (fig. 14). The prototype for the general *n*-rayed rosette almost certainly consisted of a 6-pointed star surrounded by six regular hexagons (fig. 15). This configuration becomes automatically incorporated in the pattern of 6-pointed stars described above (fig. 8), but the prototypical 6-rayed rosette seems to have been used for the first time as a distinct motif on the Arab-Ata mausoleum (978) at Tim, in Uzbekistan[30] (fig. 16), and the same pattern reappears, with slightly different proportions, on the earlier of the two Kharraqan tomb towers.[31] A method of constructing the isolated 6-rayed rosette can be derived from the construction used to produce the original pattern of 6-pointed stars (fig. 17). The centers of the (six) surrounding *peripheral stars* form the vertices *A* of a *limiting polygon* of the rosette, and circles centered on these vertices, with a radius *AB*, equal to half the edge length of the limiting polygon, determine the positions of points such as *C*, and hence the radius *CD* of the interior star of the rosette. The hexagons may be termed the *outer cells* of the rosette; the terminal segments *a* of these outer cells are obtained by drawing straight lines between such points as *B* and *E*, while the sides *b* of the outer cells are obtained by drawing lines through points such as *C*, parallel to a principal radius *BD* of the rosette. In such a case the outer cells of the rosette may be said to possess *collinear* terminal segments, and *parallel* sides. Alternatively the rosette itself can be described as a parallel-sided rosette with collinear terminal segments.



Figs. 14-15. 12- and 6-rayed geometrical rosettes.

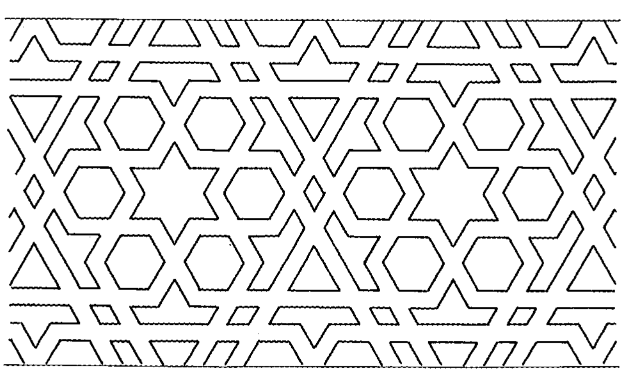
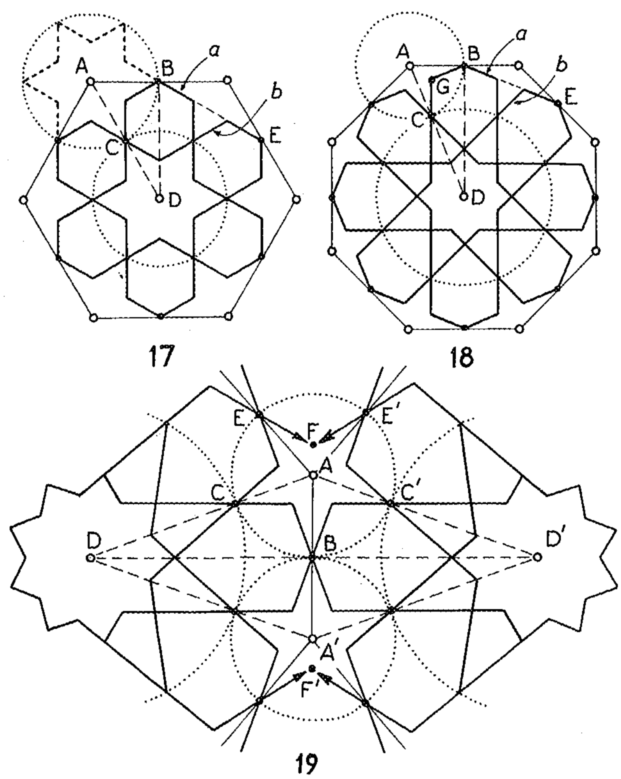


Fig. 16. The earliest occurrence of a 6-rayed rosette as a discrete motif (late 10th century onwards).

The construction just given has purposely made no reference to 6-fold symmetry or to absolute angle sizes, so it is capable of immediate generalization to include geometrical rosettes with any number of rays (fig. 18), provided only that the limiting polygon is a regular polygon. A geometrical rosette of this kind will automatically be produced, with collinear terminal segments and parallel sides. In addition, although it is not immediately obvious, segments *a* and *b* are *always equal* (this can be proved quite easily). The outer cells of the general *n*-rayed rosette are always symmetrical hexagons, but only when $n = 6$ are they regular.

When *n* is greater than 6, the geometrical rosette is nearly always constructed so as to include a 2-segment star, as defined above, in the circle with radius *CD* (fig. 18). This may be termed the *outer star* of the rosette, and this in its turn contains the *inner* or *central star.* With regard to constituent polygonal areas, the *n*-rayed rosette now consists of *n* hexagonal outer cells, *n* kite-shaped *middle cells* (or midcells), and an *n*-pointed central star. Since the angle *BAC* is greater when *n* is greater than 6 (cf. figs. 17, 18) it is obvious that exactly regular 6-pointed peripheral stars are only possible when $n = 6$. For higher values there is not enough room for a perfectly regular 6-pointed peripheral star, but a regular 5-pointed peripheral star only becomes possible when $n = 10$. Furthermore, since the angle between a pair of adjacent sides of any regular polygon is never 180°, it is obvious that regular 4-pointed peripheral stars are impossible, with the properties given above. Thus, 5-pointed peripheral stars inevitably come to be included in many patterns containing geometrical rosettes, and although they can be



Figs. 17-19. The formal construction of the geometrical rosette, generalized from the 6-rayed example in fig. 17.

regularly formed in relatively few cases, ideally they are made as regular as is possible without reducing the symmetry of the main rosettes. The highest degree of regularity is achieved as follows: the outer points of a peripheral star must lie on a circle, the center of which is a vertex of the limiting polygon of the parent rosette; the angles at its outer points should be equal; the sides or segments bordering all outer points should be equal; and the peripheral star should form collinear links with the outer star of its parent rosette(s), and with neighboring peripheral stars. Furthermore, in this most regular form, the bisectors of its outer angles meet at a single point, which may be termed the center of the peripheral star, since it coincides with the center of the circumscribing circle.

If a pair of peripheral stars is shared between two equal rosettes which are joined by a collinear link (fig. 19), then it is obvious that not only do the rosettes share an outer point B, but their limiting polygons share an edge AA'; in other words the edge length of the two limiting polygons is the same. In fact, a pair of linked rosettes could be constructed on the basis of a pair of regular polygons sharing an edge, and indeed the construction is easily generalized to include different kinds of regular polygons, and therefore rosettes of different sizes, but in this case only one of the rosettes can be constructed strictly according to the principles we have established so far. The slopes of the various line segments of the other rosette are then largely determined by those in the first rosette, but the sides and terminal segments of the outer cells of the second rosette can still be constructed as equal lengths, if required, by noting that the shoulder, or subterminal point (G, fig. 18) should always lie on the bisector of angle BAC. In order to produce repeating patterns with geometrical rosettes one might then search for tessellations or open arrangements of regular polygons in which pairs of adjacent polygons share an edge. These are the essentials of what might be termed the "polygons in contact" (PIC) method of constructing certain Islamic star patterns that was first enunciated as a general principle by E. H. Hankin,[32] although it was occasionally used earlier by Bourgoin.[33] However, in most cases it is not necessary to draw a complete arrangement of limiting polygons since the construction of just one shared edge is sufficient to determine the relative circumradii of a pair of adjacent rosettes, and subsequently the radii of their outer stars. In fact there are other exact constructions possible which achieve the same result without using the limiting polygons at all.

The earliest patterns with geometrical rosettes were mostly constructed with the properties, if not the methods, outlined above, i.e., with collinear terminal segments and parallel sides. Apart from the case when $n = 6$, rosettes are completely absent from the extremely rich ornamentation of the two Kharraqan tomb towers (1067-68 and 1093, respectively), but geometrical rosettes with 10 rays occur in the north dome chamber of the Masjid-i Jamiʿ, Isfahan (1088).[34] Rosettes with 8, 11, 12 and 16 rays occur in various patterns round the mihrab of the mosque at Barsian, Isfahan (1134)[35]—and, incidentally, include the earliest example of a remarkable pattern with 4-, 5-, 6-, 7-, and 8-pointed stars.[36] Patterns with 8- and 10-rayed rosettes occur on wooden minbars from the mosque of Ala al-Din at Konya (1155),[37] and from the Aqsa Mosque at Jerusalem (1168).[38] It seems possible, however, that the 6-fold case was generalized to include 8-, 10-, and perhaps 12-rayed rosettes well before the end of the eleventh century, and that the majority of early examples have simply not survived. A more thorough search of the relevant literature may bring to light many more dated examples from this early period.

A variety of the decagonal pattern mentioned earlier, in which the (10/3)2 stars are replaced by 10-rayed rosettes (fig. 20), is one of the most widespread of all Islamic star patterns, and is included in practically every account of these patterns published by Western authors. Reducing both to a pattern of 10-fold star-centers (fig. 21), it is evident that they share the same underlying grid of 72°/108° rhombuses. In a formal sense this may be derived by distortion from a grid of squares, or, as a tiling of 54°/72°/54° isosceles triangles, by a similar distortion from the regular tiling of equilateral triangles, but it is unlikely that either pattern was originally composed in this way.[39] Many other pattern varieties were devised on this same basis of 10-fold star-centers, and it is therefore convenient to be able to distinguish these varieties by some simple notation. The version with (10/3)2 stars (fig. 13) may be designated a type I pattern, and the stars themselves as type I motifs, since stars of this kind were the first to appear. The pattern with geometrical rosettes (fig. 20) may then be referred to as type II.

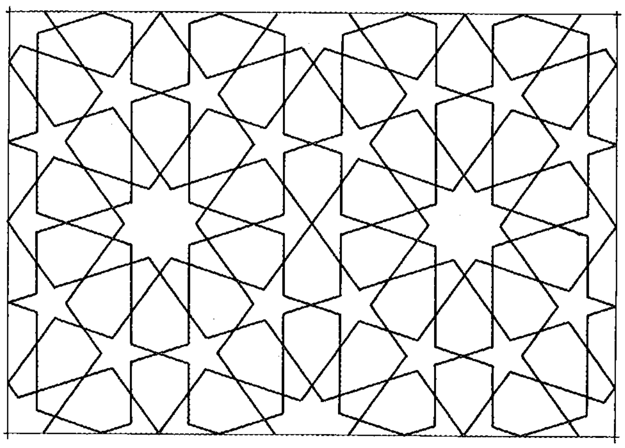


Fig. 20. 10-rayed rosettes (Type II motifs) on a grid of 72°/108° rhombs.

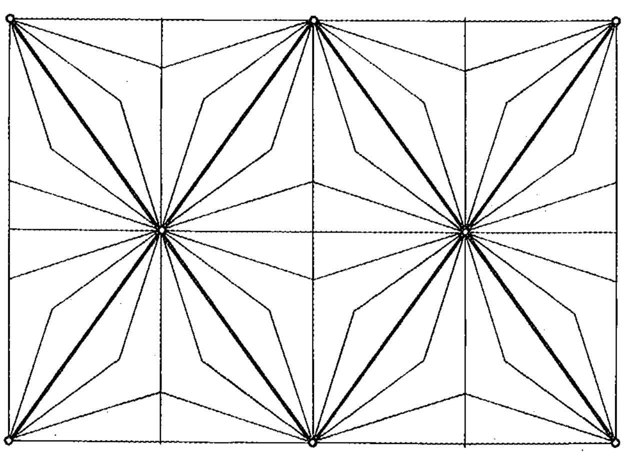


Fig. 21. Part of the grid of 72°/108° rhombs, with the space round each vertex divided into equal angles of 18°.

Both varieties display an economy of pattern shapes, with respectively four and five different kinds (not counting half shapes which inevitably occur at the edges of any finite panel), and the peripheral elements are either regular pentagons or consist of the outer shells of regular pentagrams. In fact if the two patterns are superimposed so that the centers of the star motifs coincide, it will be discovered that the vertices of the peripheral elements, pentagons and pentagrams, also coincide.[40] Furthermore, if the outer stars of the type II rosettes are reduced to the form (10/4)1, the two types become topologically equivalent in that the vertices of the peripheral elements may be regarded as fixed, or "nodal" points through which the slopes of the pattern lines are infinitely variable. In this way either type may be transformed into the other, and the varieties illustrated simply become special cases in an infinite series (this may in theory be extended even further to produce still greater variety).

One other such special case from this series is commonly used in Central Asian ornament (fig. 22), and this may be designated a type III pattern. Here the peripheral stars have their sides parallel, in pairs. An important feature of this series is that there are always two interstitial cells which are congruent to the outer cells of the star motif.

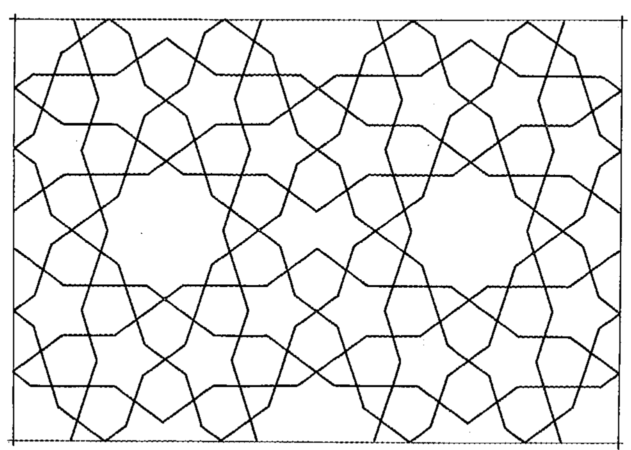


Fig. 22. A pattern with 10-fold motifs of Type III.

Unfortunately it is not possible to enter into constructional and other details here, so further pattern types on the same basis can only be briefly described and illustrated. Types IV (fig. 23) and V (fig. 24) are simply elaborations of type I patterns and are confined to Central Asia. Type VI (fig. 25) is also derived from a type I, this time by expanding each kite-shaped cell until it overlaps its neighbors in small rhombs. Initially the size of the expanded cells can be arbitrarily chosen, but in certain derivatives of this type the relative sizes of the pattern cells become rigidly determined. A frequent addition to this pattern is the incorporation of type II rosettes at the center of each type VI motif (see fig. 25, right side). Types VII (fig. 26) and VIII (fig. 27) are also Central Asian and are ultimately derivable from elements of types I and III and their later elaborations. In type VIII all star-centers are surrounded by regular decagons, but only alternate centers of type VII are so surrounded. The interiors of the decagons are variably treated in authentic sources. The fact that type VII patterns simultaneously contain two kinds of centers means that either kind can be given prominence in a small pattern area, creating quite different effects. The same remarks apply to types IX (fig. 28) and X (fig. 29), alternate centers of which employ regular decagons and 20-gons, respectively. These last two, and many related patterns, are almost exclusively used in wooden lattice work. Types XI (fig. 30) and XII (fig. 31) again form a related pair of patterns, with similar peripheral elements. Other varieties are possible, but these twelve types are the most common variants.

In addition there are many derivative variations, some of which are described below. There is some justification in giving distinct designations to these variations, since analogous treatments were applied, or can be applied, to many other arrangements using star motifs of different sizes. Similar variations may therefore be given the same type designation, irrespective of differences in the numbers of points in the constituent star motifs.

The pattern of 10-fold star-centers which underlies all the foregoing pattern types is formally related to a group which shares the same structural basis as the 72°/108° rhombus, with respect to the pattern of principal and secondary radii within the rhombus. The two axes or diagonals of a rhombus divide it into four equal right-angled triangles (fig. 21) and in the present case each has, in addition to the right angle, interior angles of 54° and 36°. Radii from the star-centers on these two vertices divide their respective angles into three

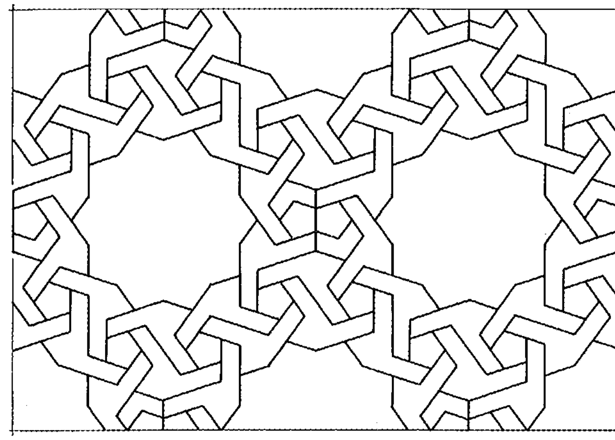


Fig. 23. A pattern with 10-fold motifs of Type IV.

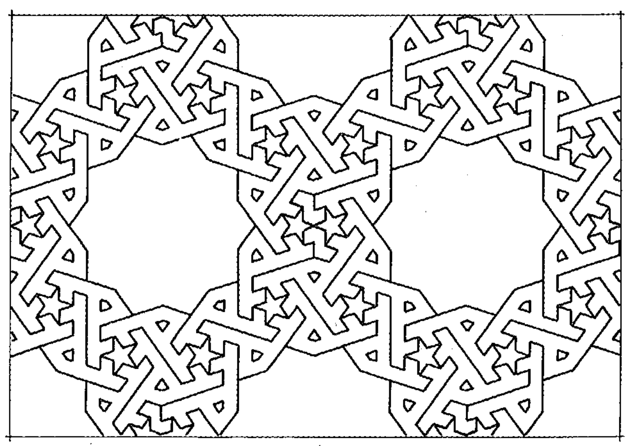


Fig. 24. A pattern with 10-fold motifs of Type V.

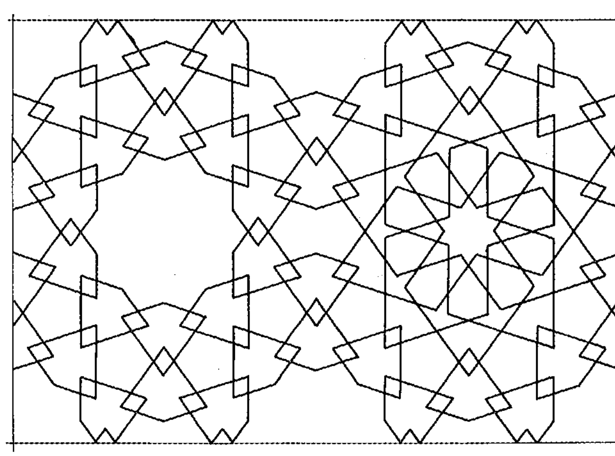


Fig. 25. A pattern with 10-fold motifs of Type VI. On the right a Type II rosette is inserted.

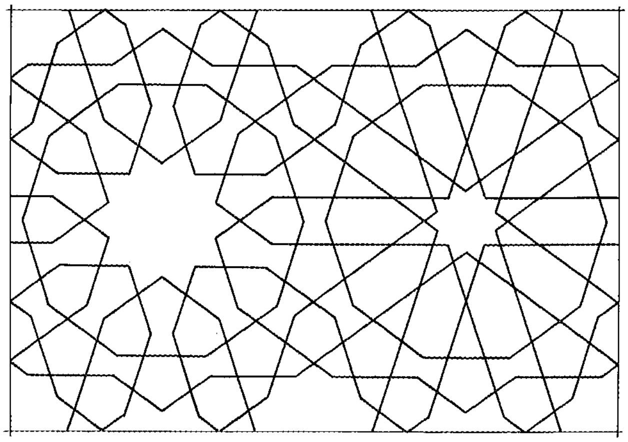


Fig. 26. A pattern with 10-fold motifs of Type VII. Motifs are of two kinds, one of which is surrounded by complete decagons. Authentic treatment of the latter is variable, as shown above.

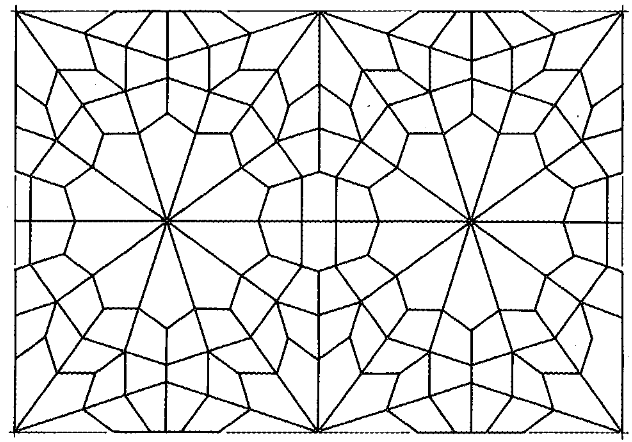


Fig. 28. A pattern with 10-fold motifs of Type IX. This and the following pattern are usually executed in wooden lattice.

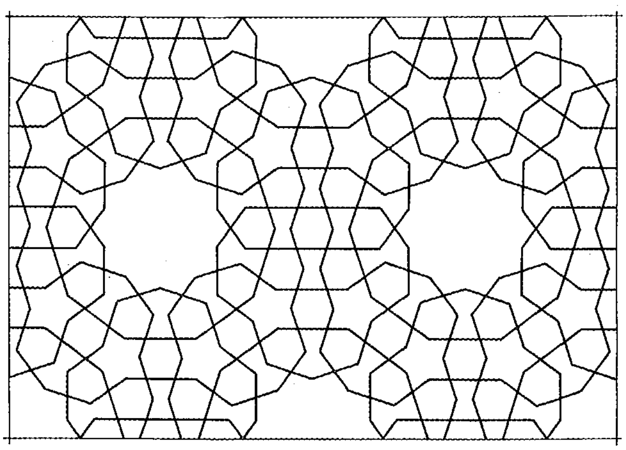


Fig. 27. A pattern with 10-fold motifs of Type VIII. All motifs are surrounded by decagons.

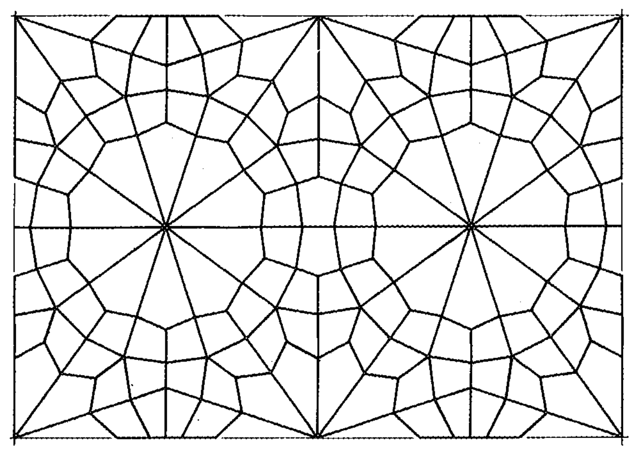


Fig. 29. A pattern with 10-fold motifs of Type X. As in fig. 28, motifs are of two kinds. Here, one kind is surrounded by complete 20-gons.

and two divisions (fig. 32), each division of 180°/10, or 18°. Let us label the vertex with three divisions the m-center, and that with two divisions the n-center. These two symbols represent the number of principal radii in each complete star-center, so that initially $m = n = 10$. This number may be termed the *star number* (or rosette number). Let p then represent the number of interradial divisions contributed by the m-center, and q the number of divisions at the n-center, so that $p = 3$ and $q = 2$. These quantities, however, may be regarded as specific values of a general relationship between the four variables m, n, p. and q. If we express the angles at m and n as fractions of 180° they become respectively p/m and q/n; the right angle is obviously 1/2. As fractions of 180° these three angles will therefore sum to unity, i.e. $p/m + q/n + 1/2 = 1$, from which we obtain the relationships $m = 2np/(n - 2q)$ and $n = 2mq/(m - 2p)$.

We are interested in cases where rhombs might occur which are structurally similar to the decagonal example previously considered, i.e., in which $p = 3$, $q = 2$. Substituting these values in the two expressions above we obtain two equations which can be solved for possible pairs of integral values of m and n, one of which must obviously be $m = n = 10$. In fact the number of possible pairs is finite, and there are only eight integral solutions. These are 30,5; 18,6; 14,7; 12,8; 10,10; 9,12;

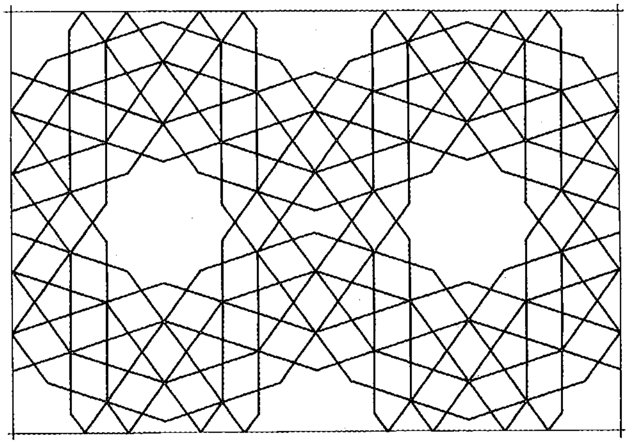


Fig. 30. A pattern with 10-fold motifs of Type XI.

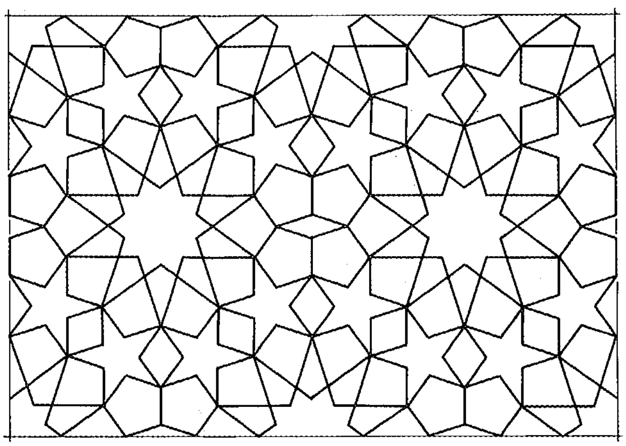


Fig. 31. A pattern with 10-fold motifs of Type XII.

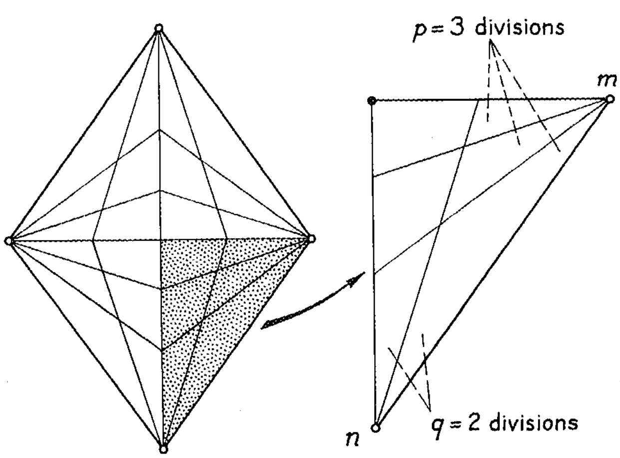


Fig. 32. Division of the 72°/108° rhomb into equal angles of 18°, producing a (3 × 2) rhomb.

8,16; and 7,28 (in each case the *m*-center is the first of the pair of values).[41]

These solutions represent in effect a series of rhombs in which *m*-fold and *n*-fold centers occur in opposite pairs on alternate vertices of each rhombus. We cannot immediately build up repeating patterns with these rhombs, however, since none of them will form a simple grid like the pattern of 10-fold star-centers with which we began. But patterns incorporating all except the first and last solutions have been used in authentic ornament, although only four of them—18,6, 12,8, 10,10 and 9,12—can be used by themselves to form repeating patterns; the others require additional shapes. The beauty of this series is that similar variations, based on the decagonal types I-XII that we have briefly described above, can be adapted to each rhombus in the series. Theoretically, one might therefore expect at least 96 distinct patterns from this series alone, but in cases where the resulting star-motifs are too dissimilar in size the patterns are often not very satisfactory.

Using the same symbols as before, we may refer to series of $(p \times q)$ rhombs, and the specific solutions of such a series may be expressed in the form $(p \times q)m, n$. Thus, the rhombs in the grid shown in fig. 21 become $(3 \times 2)10,10$ rhombs. The general notation for particular pattern types in the (3×2) rhomb series may be written as $(3 \times 2)m,n/\mathrm{T}$, where T stands for one of the twelve distinct types given above. Each rhombus in the pattern of fig. 20 thus becomes $(3 \times 2)10,10/\mathrm{II}$. Strictly speaking, of course, this notation ought to refer to the right-angled triangle which constitutes one quarter of a rhombus, but it is natural to extend it to include not only the complete rhombus, but also if necessary the two kinds of isosceles triangles which can be produced from two such identical right triangles. The context will make it clear to which we are referring at any particular time. It must be pointed out that the notation developed so far does not designate a repeating pattern, but merely a potential elementary unit for one or more repeating patterns. Additional symbols are required to indicate ways in which such elementary units are incorporated in repeating patterns, but this is unfortunately a question which cannot be pursued here.

As we have remarked above, the decagonal types I-III form special cases of an infinitely variable series, in which the two interstitial cells are congruent to the outer cells of the star motifs in each case. A similar property is characteristic of other (3×2) rhombuses with dissimilar star numbers, but in these cases the interstitial cells can be made congruent to the outer cells of only one of the two motifs, namely the m-motifs (although this congruence was rarely achieved in authentic patterns). This is an inescapable consequence of the underlying geometry of this series of rhombs, and it is capable of rigorous geometrical proof (a similar geometry is exhibited by the (2 × 1) rhomb series). Fundamental distinctions of this nature between the m- and n-centers of the (3 × 2) and other rhomb series were seldom understood by the original artists, and have certainly not been appreciated by Western authors. Among a number of lines of evidence leading to this conclusion, we may cite the many differently constructed versions of patterns using the (3 × 2)12,8/II rhombus in existence (the "rhombus" being in fact a square), some of which indicate that the artists concerned had no idea of the "correct" method of construction.[42] The PIC method will certainly allow the correct relative sizes of different star motifs to be achieved,[43] and this method is essential for type I patterns with dissimilar stars, but it is not appropriate for all twelve variations dealt with above, and there are even certain rosette patterns in which it cannot be used.

Among other rhombs in the (3 × 2) series which become incorporated in the more common patterns, we may mention (3 × 2)12,8/II, which occurs in a variety of versions from India to the Maghrib, as does (3 × 2)9,12/II. Type I patterns using these two rhombs occur from India to Egypt, but seem to be absent from the Maghrib. (3 × 2)8,16/II is commonly incorporated in Maghribi patterns but is absent or rare elsewhere; (3 × 2)8,16/I, on the other hand, seems to appear occasionally in Iran, but is entirely absent elsewhere. (3 × 2)12,8 types VII and VIII are common in Central Asia, but do not appear in other areas. It is, however, difficult to compile an inventory of pattern types throughout Islam from published sources, since it is extremely rare to find a work which illustrates every pattern occurring on a single monument, let alone a whole geographical area or historical period. Of recent publications notable exceptions are the work of Stronach and Young referred to above, and that of the Erdmanns on thirteenth-century Anatolian caravanserais.[44]

Variations such as the types I-XII we have been considering do not of course constitute all possible sources for variety in patterns which use (3 × 2) rhombuses. An extremely common derivative type I pattern is shown in fig. 33. Here the stars are separated by a "twinned pentagon" shape, forming a linear sequence which also occurs along the short axis of the original type I rhombus (fig. 13). One could continue this process, taking linear sequences along the axes of the rhombus of a basic pattern type and using the sequence obtained as the edge of a new, larger rhombus of the same shape (or of a different shape: there are two kinds of rhombs possible with 10-fold star-centers, which may be distinguished as (3 × 2) and (4 × 1) rhombs, using the present notation). However, although this is a very fruitful method for generating derivative patterns, and may indeed have been used on occasion by Muslim artists, there are many similar patterns which cannot be derived in this way.

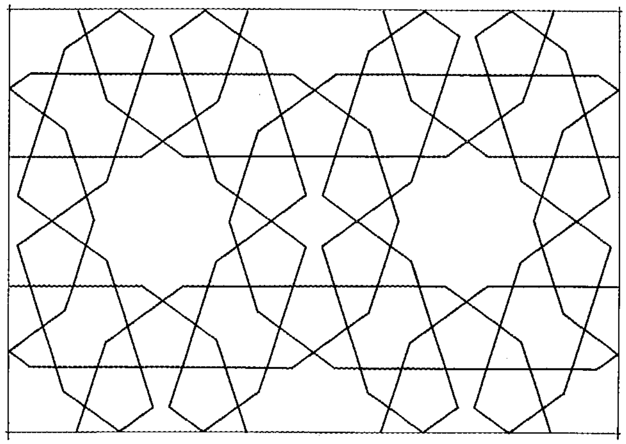


Fig. 33. A derivative pattern of 10-pointed stars, formed by rearrangement of elements in fig. 13.

A different source of variety makes use of parallel links (fig. 34). Only one example is illustrated, but it is possible to classify such patterns in many different categories; there is no end to this kind of variation. Of even greater interest, it is possible to imitate many of these derivative decagonal patterns in other rhombs of the (3 × 2) series, especially in the case of (3 × 2)12,8 and (3 × 2)9,12, since in these there is very little distortion of the pentagons, pentagrams, and other shapes of the decagonal patterns. A number of such derivatives exist as authentic patterns throughout Islam.

In the limited space available it has been possible to give no more than a glimpse of some of the methods by means of which these star patterns can be investigated. There are of course many numerical procedures which can be used to investigate different combinatorial aspects of star-pattern construction beyond those briefly mentioned above. We have dealt with only rhombic configurations of star-centers—and even here,

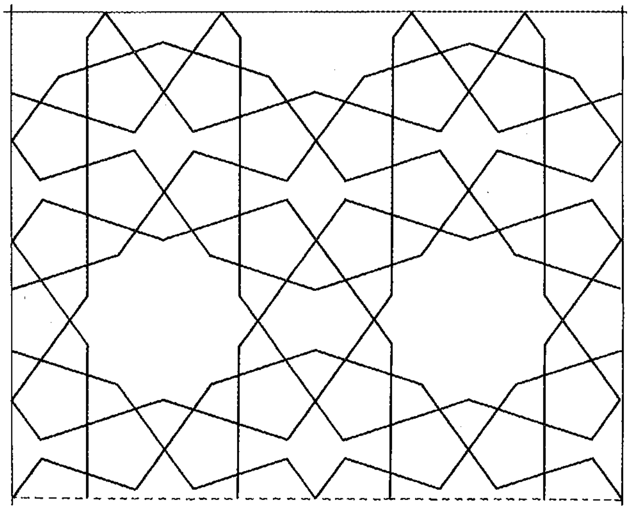


Fig. 34. A further derivative pattern of 10-pointed stars, in which some stars are joined by parallel links. The underlying grid is rhombic, but the rhomb angles are no longer simple multiples of 18°.

with only one series, the (3 x 2) series—and yet we have not described ways in which rhombs of one or more kinds can be arranged to build up repeating patterns. This would entail a classification of rhombic tessellations along lines somewhat different from those employed in classifying tessellations of regular polygons.

There are also many possibilities for repeating patterns using almost regular star-centers, relatively few of which exist as authentic Islamic ornament, and these also lend themselves to numerical methods of investigation. Although by no means all authentic patterns are based on rhombic tessellations, this does form a very useful approach since it accounts for most of the commoner patterns, including hexagonal and square-based arrangements as special cases. It is not suggested that the first designers of Islamic patterns used similar lines of reasoning to those outlined above, but theoretical studies of this kind are of particular value in that one can attempt a systematic and exhaustive enumeration of all possible ways of combining authentic star motifs and their variation. Since no one can possibly study the vast profusion of existing Islamic patterns at first hand in order to be able to classify all authentic varieties, a theoretical approach is clearly an advantage. It then becomes of interest to compare the results of such studies with the results actually achieved by the original Muslim artists, since the comparison may provide clues to the early craftsman's understanding of the deeper geometry of his patterns.

NOTES

1. K. A. C. Creswell, *A Bibliography of the Architecture, Arts and Crafts of Islam to 1st January, 1960* (Cairo: American University at Cairo Press, 1961), "Ornament," pp. 963-78, and Supplement (1973), "Ornament," pp. 290-91; D. Wade, *Pattern in Islamic Art* (London, 1976); I. El-Said and A. Parman, *Geometric Concepts in Islamic Art* (London, 1976); K. Critchlow, *Islamic Patterns: An Analytic and Cosmological Approach* (London, 1976). For a general appraisal of the role of ornament in Islamic art, see D. Jones, "The Elements of Decoration: Surface, Pattern and Light," in George Michell, ed., *Architecture of the Islamic World* (London, 1978).
2. Perhaps the intended collaboration between Gómez-Moreno and Prieto Vives was to have been such a work, but only part of it was ever published (A. Prieto Vives and M. Gómez-Moreno, *El Lazo: Decoracíon geométrica musulmana* [Madrid, 1921]). A great deal of unpublished material still exists from this collaboration, however, and it is to be published in the near future (see M. Gómez-Moreno, "Una de mis teorías del Lazo," *Cuadernos de la Alhambra* 10-11 [Granada, 1974-75]). For a discussion of this abortive collaboration and a list of papers published by Prieto Vives, see D. Cabanelas, "La Antigua policromia de techo de Comares en la Alhambra," *Al-Andalus* 35 (Madrid, 1970): 435-37.
3. See the discussion by J. M. Rogers, "The 11th Century—A Turning Point in the Architecture of the Mashriq?," in D. S. Richards, ed., *Islamic Civilisation 950-1150* (Papers on Islamic History 3) (Oxford, 1973), pp. 221-24.
4. Oleg Grabar (*The Alhambra* [London, 1978]), referring (on pp. 195-96) to an unpublished doctoral thesis, cites "evidence which is only now being discovered" indicating that new methods of composing geometrical patterns after the tenth century, emphasizing polygons and stars, were "made possible by a conscious attempt on the part of professional mathematicians and scientists to explain and to guide the work of artisans." It is not yet clear whether this guidance refers to a choice of motifs or their construction, or to methods of combining motifs in repeating patterns, or whether perhaps the mathematicians themselves are supposed to have designed new patterns effectively. Even if this claim is true, the influence of the mathematicians cannot have been very widespread or long lasting, judging by the many clumsily constructed patterns in existence, often employing widely varying and sometimes arbitrary methods of layout, or even drawn more or less freehand.
5. R. Orazi, *Wooden Gratings in Safavid Architecture* (Rome, 1976), text in Italian and English, p. 104 and note.
6. My own unpublished research.
7. E. H. Lockwood and R. H. Macmillan, *Geometric Symmetry* (Cambridge, 1978).
8. These are (1) rotation about a point, (2) displacement in a given direction, (3) "reflection" across a straight line, and (4) a "glide-reflection," which combines (2) and (3) in one operation.
9. Although many examples of Islamic patterns occur on the curved surfaces of domes, true spherical patterns are rare; they are principally found as quarter-spheres capping the hemicylindrical niche of many mihrabs. Hemispherical bosses sometimes occur, decorated with geometrical patterns. A sphere is a real surface of constant positive curvature, and Arab mathematicians were familiar with its geometry. The hyperbolic plane on

the contrary is an infinite imaginary surface of constant negative curvature (i.e., it is everywhere saddle-shaped) discovered by European mathematicians in the early nineteenth century. It is possible to map the whole hyperbolic plane inside a circle, and in this form many interesting tessellations have been made visible. The Dutch artist Maurits Escher adapted many of his bizarre tessellations to this circular representation. See J. L. Locher, ed., *The World of M. C. Escher* (New York, 1971). No one seems yet to have attempted to adapt Islamic patterns to the hyperbolic plane.

10. The class of tilings I have described here have been termed edge-to-edge tilings. However, other types exist in which, for example, one of two vertices limiting an edge of one tile lies part way along an edge of an adjacent tile. Some Islamic tile patterns are of this type.
11. If the interlacing bands form only closed circuits or infinite lengths without any loose ends, it can be proved that the crossovers can always be arranged so that along any band they run alternately over and under other bands; two adjacent crossovers of the same type need never occur. Thus the artist has no need to worry that his interlacing pattern will not "work out," provided he does not start the crossovers simultaneously in more than one place. The interlacing principle is linked to certain topological properties of the plane, and there are other mathematical surfaces on which it does not hold. See H. A. Thurston, "Celtic Interlacing Patterns and Topology," *Science News* 33 (1954): 50-62.
12. In the tessellation of equilateral triangles, the vertices constitute hexads, the centers of the triangles triads, and the midpoints of the edges diads. Any stars can be placed on these rotocenters in which the numbers of points are multiples of 6, 3, and 2 respectively. Similarly the tessellation of squares can accommodate numbers that are multiples of 4, 4, and 2 respectively. Most, if not all, of the simpler permutations of this kind are encountered in authentic Islamic patterns.
13. J. M. Rogers, "The 11th Century" (cited above, n. 2).
14. K. A. C. Creswell, *A Short Account of Early Muslim Architecture* (Harmonsworth: Penguin Books, 1958), pp. 75 ff., and pl. 16.
15. M. Selim Abdul-Hak, "La reconstitution d'une partie de Kasr al-Heir al-Gharbi au Musée de Damas" (in Arabic), *Les Annales Archéologiques de Syrie* 1(1)B (Damascus, 1951), pp. 5-57, pls. 15-24.
16. R. W. Hamilton, *Khirbat al-Mafjar* (Oxford, 1959).
17. B. Pavón Maldonado, *El Arte hispanomusulmán en su decoracion geometrica* (Madrid, 1975).
18. E. Herzfeld, *Die Ausgrabungen von Samarra* I. *Der Wandschmuck der Bauten von Samarra und seine Ornamentik* (Berlin, 1923), fig. 234, house 3, room 16.
19. K. A. C. Creswell, "Some Newly Discovered Tulunid Ornament," *Burlington Magazine* 35 (1919): 180-88, pl. II G.
20. Pavon Maldonado, *Arte hispanomusulmán*, pl. 31.
21. K. A. C. Creswell, *Muslim Architecture of Egypt* (Oxford, 1952), vol. 1, fig. 29 and pl. 26*e*.
22. M. S. Abdul-Hak, "Reconstitution d'une partie de Kasr al-Heir," pl. 19 (bottom).
23. D. Whitehouse, "Excavations at Siraf: Third Interim Report," *Iran* 8 (1970): 1-18, pl. 11a.
24. Some authors would perhaps consider this a moot point. These crafts tend to remain conservative, the artisans continuing to use traditional formulae and resisting any sudden innovations. It is therefore conceivable that the initial idea for generalizing these star patterns to include stars with any number of points came from professional mathematicians, or at least from someone with sufficient imagination to glimpse something of the possibilities inherent in this kind of ornament. If so, this initial push must have occurred sometime toward the end of the tenth century and probably no later than the first half of the eleventh. Even granted such an external stimulus, however, the solutions subsequently discovered follow quite logically and inevitably, and it is entirely possible that the artisans themselves could then have designed the definitive patterns, using trial-and-error methods.
25. D. Stronach and T. C. Young, "Three Seljuq Tomb Towers," *Iran* 4 (1966): 1-20, pl. 9a.
26. Ibid., pl. 4b.
27. A. U. Pope, "Note on the Aesthetic Character of the North Dome of the Masjid-i Jami of Isfahan," in *Studies in Islamic Art and Architecture in Honour of Prof. K. A. C. Creswell* (Cairo: American University in Cairo Press, 1965), pp. 179-93; fig. 7.
28. Fairly clear pictures of this ornament are given in D. Hill and O. Grabar, *Islamic Architecture and Its Decoration* (London, 1964), figs. 146-48.
29. J. Bourgoin, *Les Éléments de l'art arabe: le trait des entrelacs* (Paris, 1879), reissued by Dover Publications as *Arabic Geometrical Pattern and Design* (New York, 1973).
30. G. A. Pugachenkova, "Mavzolei Arab-Ata," *Iskusstvo Zodchikh Uzbekistana* II (Tashkent, 1963), figs. 16, 17.
31. Stronach and Young, "Three Seljuq Tomb Towers," pl. 4b.
32. E. H. Hankin, "On Some Discoveries of the Methods of Design Employed in Mohammedan Art," *Journal of the Society of Arts* 53 (1905): 461-72; idem, "The Drawing of Geometric Patterns in Saracenic Art," *Memoirs of the Archaeological Survey of India* 15 (Calcutta, 1925).
33. Bourgoin, *Éléments de l'art arabe*, pls. 123, 124.
34. U. Vogt-Goknil, *Mosquées* (Paris, 1975), p. 78.
35. M. B. Smith, "Material for a Corpus of Early Iranian Islamic Architecture, II: Manar and Masdjid, Barsian (Isfahan)," *Ars Islamica* 4 (1937): 7-41, figs. 25, 27.
36. Ibid., fig. 27. Another design on the same basis is shown in Bourgoin, *Éléments de l'art arabe*, pl. 163, and appears to have been taken from paired window grilles on the main façade of the mosque of Sarghatmish in Cairo. Yet another version occurs on the mausoleum of Mumineh Khatun (1186-87) at Nakhichevan in southern Azerbaijan (see M. A. Useinov, *Pamyatniki Azerbaidjanskogo zodchestva* [Moscow, 1951], central face of plate 5). The limiting polygons of the stars or rosettes of these patterns form a tessellation of almost regular polygons, which is the basis for E. Holiday's first series of Altair Design sheets (Altair Design [London: Longman Group, 1970]).
37. O. Aslanapa, *Turkish Art and Architecture* (London, 1971), fig. 20.
38. E. Herzfeld, "Damascus: Studies in Architecture, IV," *Ars Islamica* 13-14 (1948): 118-38, fig. 24.
39. For a contrary view, cf. B. Pavón Maldonado, *Arte hispanomusulmán*, pp. 338-39, and fig. 85, drawings 5 and 6.
40. A fact known to some fourteenth-century artists, since types I and II are used superimposed as decoration in Uljaytu's mausoleum at Sultaniyya, Iran. See A. U. Pope, ed., *A Survey of Persian Art* (London, 1939), vol. 4, pl. 383.
41. Curves can be drawn from these equations in order to find pairs of values for other kinds of rhombs. The values which are of interest lie on the positive arms of hyperbolas, with asymptotes

at m = $2p$, n = $2q$. Each series has a pair of values for which $m = n = 2(p + q)$.

42. In a sense the construction of many Islamic patterns is arbitrary until rules have been formulated defining the degrees of regularity one wishes to achieve in each part or in the pattern as a whole. Since rules of this kind were never explicitly stated by the Muslim artist, it might be thought irrelevant to describe authentic examples as "correctly" or "incorrectly" constructed. Nevertheless it seems evident that the majority of artisans were striving for the highest level of symmetry and regularity attainable in each pattern, and that the proportions of the parts within the patterns were adjusted accordingly. Using the principle of the greatest possible symmetry and regularity as a basis for comparison, it becomes feasible to define an "incorrectly" constructed pattern as one which falls short of achieving the highest degree of symmetry possible for that particular pattern. Unfortunately one cannot appeal to authentic patterns as the final arbiter, since there is rarely a unique and universally adopted construction of any given pattern.
43. The relative radii of two dissimilar stars constructed according to the PIC method are in a precise ratio, the numerical value of which can be calculated to any desired accuracy by trigonometrical or other means. However, it is a curious fact that the more nearly equal are the two values m and n, the closer the respective radii are to the ratio $m : n$. So close are some of these approximations to the theoretical values that it becomes impossible to distinguish between the theoretical and $m : n$ ratios from measurements on authentic patterns.
44. K. Erdmann and Hanna Erdmann, *Das anatolische Karavansaray des 13. Jahrhunderts*, pts. 2-3 in one vol. (Berlin, 1976), *Die Ornament*, pp. 109-205.